

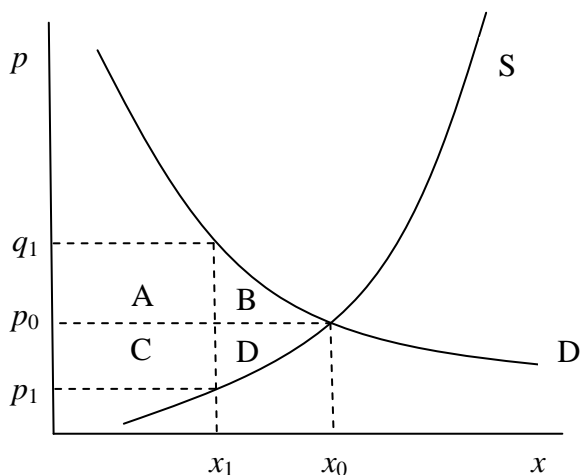
## Economics 230a, Fall 2016

### Lecture Note 6: Basic Tax Incidence

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Tax incidence refers to where the burden of taxation actually falls, as distinguished from who has the legal liability to pay taxes. As with deadweight loss, it is a concept for which the intuition is clear, but for which actual measurement requires the specification of a precise conceptual experiment. It is not enough simply to ask, “What is the incidence of a tax on good  $x$ ?” We must specify what is done with the revenue, i.e., whether it is (1) spent in a way that has no further effects on welfare (absolute incidence); (2) spent on goods and services, which also have an impact on welfare (balanced-budget incidence); or (3) used to reduce other taxes (differential incidence).

To illustrate the concept of incidence (in this case, absolute incidence), consider a small tax introduced in some competitive market, in which the initial price is  $p_0$  and the initial quantity  $x_0$ . We introduce a tax, which reduces output, increases the consumer price  $q$ , and reduces the producer price  $p$ , in the manner shown below. For simplicity, we will assume that the revenue is spent by the government in the same manner that the consumer would spend it. Thus, total demand (by the consumer plus the government) is the same as it would be if the consumer were given the revenue. Starting at an undistorted equilibrium, this is roughly equivalent to compensated demand, since there is no first-order deadweight loss.



The burden of the tax falling on the demand side is the loss of consumer's surplus  $A+B$ , while the burden on the producer is the loss of producer's surplus  $C+D$ , the sum exceeding revenue  $(A+C)$  by the deadweight loss  $B+D$ . For a small change starting at a Pareto optimum, the first-order excess burden is small relative to the revenue cost and we can approximate burdens by  $x\Delta q$  for the consumer and  $-x\Delta p$  for the producer, with the total burdens equal to revenue in this first-order approximation.

The relative burdens on the demand and supply sides will depend on relative elasticities. Defining the term  $\hat{z} = d \log(z)$ , and letting the demand and supply elasticities (defined to be non-negative) be  $\eta^D$  and  $\eta^S$ , we know that  $\hat{x} = -\eta^D \hat{q} = \eta^S \hat{p}$ . Further, if we let  $T = (1+\tau)$ , where  $\tau$  is the *ad valorem* tax imposed on the producer price, we have  $q = Tp$ , so that  $\hat{q} = \hat{T} + \hat{p}$ . (Also, assuming that we are starting at a value of  $\tau = 0$ ,  $\hat{T} = d\tau$ .) Thus, setting the two expressions for  $\hat{x}$  equal we have  $-\eta^D (\hat{T} + \hat{p}) = \eta^S \hat{p} \Rightarrow \hat{p} = \frac{-\eta^D}{\eta^D + \eta^S} \hat{T}$ ;  $\hat{q} = \frac{\eta^S}{\eta^D + \eta^S}$ ; the ratio of the shares of the burden on consumers and producers is  $\eta^S/\eta^D$ , i.e., is proportional to the inverse ratio of the respective elasticities – the greater the responsiveness, the lower the burden.

Note: it does not matter whether the tax is imposed on the buyer or the seller, assuming that prices are flexible.

### Application: The Berkeley Soda Tax

Many taxes on specific goods are “sin” taxes, on items such as tobacco and alcohol. Such taxes may be desirable if there are no externalities, if individuals have self-control problems. This was the logic that underlay the adoption of a tax on sweetened beverages by the city of Berkeley at the end of 2014, the first in the United States (Philadelphia recently adopted the second). When thinking about this tax, the question arises to what extent a local jurisdiction can have any impact on outcomes. For a very small jurisdiction imposing an excise tax, one might expect that both supply and demand elasticities would be very high, as both consumers and producers can shift to nearby jurisdictions. Thus, one would expect local purchases to fall, but the relative impact on consumer and producer prices is less obvious. The paper by Cawley and Frisvold studies the impact of Berkeley’s soda tax, using San Francisco and diet beverages as controls, and finding that less than half of the tax was passed on to consumers.

But, if the incidence of the soda tax fell partially on suppliers, this still leaves unresolved whether this fell on profits, wages, rents, etc. To analyze incidence more fully in terms of factor incomes, we introduce a simple, two-sector general equilibrium model that is a standard tool for incidence analysis.

### The Harberger Model

Assumptions:

- Two factors of production,  $K$  and  $L$ , in fixed overall supply,  $\bar{K}$  and  $\bar{L}$ .
- Two competitive sectors of production,  $X$  and  $Y$ , with CRS production functions
- One representative consumer who spends factor income on the two goods
- Starting from an undistorted equilibrium, government raises tax revenue and spends it in exactly the same way the household would

The last assumption implies that for small taxes the changes in total (household plus government) demand will lie along the household’s initial indifference curve, because there is no first-order deadweight loss.

### Basic Equations

By definition,

$$(1) \quad \hat{X} - \hat{Y} \equiv -\sigma_D(\hat{q}_X - \hat{q}_Y),$$

where  $\sigma_D$  is the demand elasticity of substitution (defined to be non-negative) and  $q_i$  is the consumer price of good  $i$ . Also, as a consequence of cost minimization by producers, the derivative of the cost function with respect to the price of a factor is the quantity of that factor used in production; competition implies that price equals marginal cost. It follows that for each production sector  $i$ ,  $\hat{p}_i = \theta_{Li}\hat{w} + \theta_{Ki}\hat{r}$ , where  $w$  and  $r$  are the returns to labor and capital and  $\theta_{ji}$  is the share of payments to factor  $j$  in sector  $i$ ’s costs. For example,  $\theta_{LX} = wL_X/p_XX$ , where  $L_X$  is the amount of labor used in sector  $X$ . Note that the shares  $\theta$  in each sector must sum to 1, so that

$\hat{p}_i = \theta_{Li}\hat{w} + (1 - \theta_{Li})\hat{r}$  for each sector. If we subtract this expression for sector  $Y$  from that for sector  $X$ , we get:

$$(2) \quad \hat{p}_X - \hat{p}_Y = \theta^*(\hat{w} - \hat{r}),$$

where  $\theta^* = (\theta_{LX} - \theta_{LY})$  measures the labor intensity of sector  $X$  relative to sector  $Y$ . If  $\theta^* > 0$ , the relative price of good  $X$  will rise with an increase in the wage relative to the return to capital.

Finally, we can relate factor returns to the production of goods  $X$  and  $Y$ . Intuitively, we would expect an increase in production of good  $X$  to lead to greater demand and a higher relative factor return to whichever factor sector  $X$  uses more intensively than sector  $Y$ .

By definition of the production elasticities of substitution,  $\sigma_X$  and  $\sigma_Y$ ,  $\hat{K}_i - \hat{L}_i = \sigma_i(\hat{w} - \hat{r})$  for  $i = X, Y$ . For convenience, express  $K$  and  $L$  as ratios of output, e.g.,  $k_X \equiv K_X/X$ . It follows that

$$(3) \quad \hat{k}_i - \hat{l}_i = \sigma_i(\hat{w} - \hat{r}) \quad i = X, Y$$

By the envelope theorem, we know that derivatives of the cost function satisfy  $d(rk_i + wl_i) = k_i dr + l_i dw$ , so  $rdk_i + wdl_i = 0$ . This implies that

$$(4) \quad \left( \frac{rk_i}{p_i} \right) \hat{k}_i + \left( \frac{wl_i}{p_i} \right) \hat{l}_i = \theta_{Ki} \hat{k}_i + \theta_{Li} \hat{l}_i = 0, \quad i = X, Y.$$

Finally, note that  $L_X + L_Y = l_X X + l_Y Y = \bar{L}$ ;  $K_X + K_Y = k_X X + k_Y Y = \bar{K}$ ; totally differentiating:

$$(5a) \quad (\hat{l}_X + \hat{X})\lambda_{LX} + (\hat{l}_Y + \hat{Y})\lambda_{LY} = 0; \quad \text{also} \quad (5b) \quad (\hat{k}_X + \hat{X})\lambda_{KX} + (\hat{k}_Y + \hat{Y})\lambda_{KY} = 0$$

where  $\lambda_{LX} = L_X / \bar{L}$  is the share of the economy's labor that is used in sector  $X$ , and the other terms are defined in the same manner.

Now, substitute (4) into (3) for both sectors to get expressions for  $\hat{l}_X$  and  $\hat{l}_Y$  and (using the fact that the labor and capital cost shares  $\theta$  add to 1 for each sector, and that  $\lambda_{LX} + \lambda_{LY} = 1$ ) substitute these expressions into (5a) to obtain:

$$(6a) \quad \lambda_{LX} \hat{X} + \lambda_{LY} \hat{Y} = (\lambda_{LX} \theta_{KX} \sigma_X + \lambda_{LY} \theta_{KY} \sigma_Y)(\hat{w} - \hat{r})$$

Follow the same procedure to get expressions for  $\hat{k}_X$  and  $\hat{k}_Y$  to substitute into (5b) to obtain:

$$(6b) \quad \lambda_{KX} \hat{X} + \lambda_{KY} \hat{Y} = -(\lambda_{KX} \theta_{LX} \sigma_X + \lambda_{KY} \theta_{LY} \sigma_Y)(\hat{w} - \hat{r}),$$

and subtract (6b) from (6a) to obtain:

$$(7) \quad \lambda^*(\hat{X} - \hat{Y}) = (a_X \sigma_X + a_Y \sigma_Y)(\hat{w} - \hat{r}) = \bar{\sigma}(\hat{w} - \hat{r})$$

where  $a_i (= \lambda_{Li}\theta_{Ki} + \lambda_{Ki}\theta_{Li})$  is a weighted average of sector  $i$ 's share of production, as measured by its use of labor and capital,  $\lambda_{Ki}$ , and labor,  $\lambda_{Li}$ , and  $\lambda^* (= \lambda_{LX} - \lambda_{KX})$  is positive (negative) if sector  $X$  is more (less) labor intensive than sector  $Y$ . As expected, a shift in production toward  $X$  will increase the relative return to the factor that  $X$  uses relatively intensively. The effect will be stronger the smaller is the average elasticity of substitution,  $\bar{\sigma}$ , because it will take larger changes in factor prices to induce the changes in factor intensities needed to clear factor markets.

Note that (2) and (7) combined provide an expression for the production possibilities frontier,

$$(8) \quad \hat{p}_X - \hat{p}_Y = \frac{\lambda^*\theta^*}{\bar{\sigma}}(\hat{X} - \hat{Y}). \quad (\text{Note that } \text{sgn}(\lambda^*) = \text{sgn}(\theta^*), \text{ so the frontier is convex.})$$

Equations (1), (2), and (7) are a system in four unknowns,  $(\hat{w} - \hat{r})$ ,  $(\hat{p}_X - \hat{p}_Y)$ ,  $(\hat{X} - Y)$  and  $(\hat{q}_X - \hat{q}_Y)$ . We add a fourth equation by introducing a tax. We begin with a tax on good  $X$ , setting  $q_X = T_X p_X$ , so that:

$$(9) \quad \hat{q}_X - \hat{q}_Y = \hat{p}_X + \hat{T}_x - \hat{p}_Y$$

Solving this system of equations, we obtain:

$$(10) \quad \hat{p}_X - \hat{p}_Y = -\frac{\sigma_D}{\frac{\bar{\sigma}}{\lambda^*\theta^*} + \sigma_D} \hat{T}_x; \quad \text{and} \quad (11) \quad \hat{q}_X - \hat{p}_Y = \frac{\frac{\bar{\sigma}}{\lambda^*\theta^*}}{\frac{\bar{\sigma}}{\lambda^*\theta^*} + \sigma_D} \hat{T}_x$$

Expressions (10) and (11) say that, if we take good  $Y$  as the numeraire (i.e.,  $\hat{p}_Y = 0$ ), the burden of the tax is borne on the demand and supply sides of  $X$  according to the values of terms that relate to demand and supply. As will now be demonstrated, these expressions are basically equivalent to those derived in the simple partial equilibrium example based on demand and supply elasticities.

Note that the term  $\frac{\bar{\sigma}}{\lambda^*\theta^*}$  comes from the expression for the production possibilities frontier, (8).

Under profit maximization,  $p_X dX + p_Y dY = 0 \Rightarrow \hat{Y} = -\frac{p_{XX}}{p_{YY}} \hat{X}$ , so (8) implies:

$$(8') \quad \hat{X} \left(1 + \frac{p_{XX}}{p_{YY}}\right) = \frac{\bar{\sigma}}{\lambda^*\theta^*} (\hat{p}_X - \hat{p}_Y)$$

With good  $Y$  as numeraire,  $\hat{p}_Y = 0$  and (8') may be rewritten:

$$(12) \quad \frac{\bar{\sigma}}{\lambda^*\theta^*} = \frac{\hat{X}}{\hat{p}_X} \left(1 + \frac{p_{XX}}{p_{YY}}\right) = \eta_X^S \left(1 + \frac{p_{XX}}{p_{YY}}\right),$$

where  $\eta_X^S$  is the elasticity of supply of good  $X$  with respect to its producer price. Now, consider consumer demand, which is determined by the elasticity of substitution,  $\sigma_D$ , according to (1).

Under utility maximization,  $dU = q_X dX + q_Y dY = 0 \Rightarrow \hat{Y} = -\frac{q_{XX}}{q_{YY}} \hat{X}$ , so (1) implies:

$$(1') \quad \hat{X} \left(1 + \frac{q_{XX}}{q_{YY}}\right) = -\sigma_D (\hat{q}_X - \hat{p}_Y)$$

Again using the fact that good  $Y$  is numeraire, (1') may be rewritten:

$$(13) \quad \sigma_D = -\frac{\hat{X}}{\hat{q}_X} \left(1 + \frac{q_{XX}}{p_Y Y}\right) = \eta_X^D \left(1 + \frac{q_{XX}}{p_Y Y}\right)$$

where  $\eta_X^D$  is the elasticity of demand of good  $X$  with respect to its consumer price. Substituting (12) and (13) into the incidence expression (11), and noting that  $q_X = p_X$  in the initial equilibrium, we have:

$$(14) \quad \hat{q}_X - \hat{p}_Y = \frac{\eta_X^S}{\eta_X^S + \eta_X^D} \hat{T}_X,$$

which is precisely the partial-equilibrium expression for the impact on the taxed good's consumer price.

Returning to the general incidence solution, we combine (10) and (2) to obtain:

$$(15) \quad (\hat{w} - \hat{r}) = -\frac{1}{\theta^*} \frac{\sigma_D}{\frac{\sigma}{\lambda^* \theta^*} + \sigma_D} \hat{T}_X.$$

This expression says that the tax on good  $X$ , which lowers the producer price of good  $X$ , will also lower the ratio  $w/r$  if sector  $X$  is labor intensive – a tax on the labor-intensive good is relatively bad for labor. How would we measure the share of the burden borne by labor? Intuitively, if  $w/r$  is fixed, i.e.,  $\hat{w} - \hat{r} = 0$ , then the tax is borne in proportion to each factor's share of income – since relative rates of return don't change, and factor supplies are fixed, an increase in the consumer price of good  $X$  will lower real factor incomes of labor and capital by the same proportion. More generally, we can ask what fraction,  $\psi$ , of the tax revenue we would have to give back to labor in order to keep labor's share of *gross* income (including the tax),  $\frac{wL + \psi(T_X - 1)p_X X}{wL + rK + (T_X - 1)p_X X}$ , constant. Clearly, if  $w/r$  doesn't change as the tax is imposed,  $\psi = \frac{wL}{wL + rK}$ . If  $\hat{w} - \hat{r} < (>)0$ ,  $\psi$  is larger (smaller).

Now, consider a partial factor tax on capital used in sector  $X$ , which is how Harberger conceived of the corporate income tax – as an additional tax on capital used in the corporate sector. (Note that a general tax on capital income in this model is simply borne by capital, as capital is in fixed overall supply, so the only interesting factor-tax incidence question involves the differential tax in one sector.) Intuitively, we should expect this tax to have two effects. The first will be to raise the cost of good  $X$ , just like the excise tax. (The fact that the tax is levied on the production side, rather than on the transaction with the consumer, is irrelevant.) The second will be to discourage the use of capital in production, which should shift the incidence further onto capital. These are sometimes referred to as the excise tax effect and the factor substitution effect of the partial factor tax.

To solve for the effects of this tax, we replace  $r$  with  $rT_{KX}$  in any equations involving the return to capital in sector  $X$ . Thus, we get  $\hat{p}_X = \theta_{LX}\hat{w} + \theta_{KX}(\hat{r} + \hat{T}_{KX})$ , which implies:

$$(2') \quad \hat{p}_X - \hat{p}_Y = \theta^*(\hat{w} - \hat{r}) + \theta_{KX}\hat{T}_{KX}$$

This expression picks up the excise tax effect. Also, equation (7) is modified as follows:

$$(7') \quad \lambda^*(\hat{X} - \hat{Y}) = a_X \sigma_X (\hat{w} - \hat{r} - \hat{T}_{KX}) + a_Y \sigma_Y (\hat{w} - \hat{r}) = \bar{\sigma}(\hat{w} - \hat{r}) - a_X \sigma_X \hat{T}_{KX},$$

which picks up the factor substitution effect, showing, for example, that even if  $X/Y$  doesn't change,  $\hat{w} - \hat{r} > 0$ .

Solving (1), (2'), and (7') (and using the fact that consumer prices  $q$  and producer prices  $p$  are equal – the tax is imposed on producers and hence already included in  $p$ ), we get the analogue for (15) above:

$$(15') \quad (\hat{w} - \hat{r}) = \frac{\frac{1}{\theta^*} \sigma_D \theta_{KX} + \frac{a_X \sigma_X}{\lambda^* \theta^*}}{\frac{\bar{\sigma}}{\lambda^* \theta^*} + \sigma_D} \hat{T}_{KX},$$

in which the two terms in the numerator of the right-hand side account for the excise tax effect (which can be positive or negative) and the factor substitution effect (which is non-negative). Harberger showed that under a variety of reasonable assumptions (such as all three elasticities being equal), capital bears exactly 100 percent of the tax. Note that this is the burden on *all* capital – as capital flees the corporate sector, this movement depresses capital returns in the noncorporate sector as well.

Both the realism of the Harberger model for studying corporate tax incidence and the characterization of the corporate income tax as an extra tax on corporate capital are subject to question, as discussed in considerable detail by the subsequent literature on the effects of the corporate tax, reviewed in Auerbach's survey paper.